What is a Matrix?
How do you use matrices?
Can you Enter matrices in a calulator?
How can I add and subtract matrices by hand? using a calculator?
How can I multiply a matrix by a constant value (a scalar multiple)?
All these questions and more will be answered as we explore the WORLD of MATRICES

DE Math Standards: 1,2,5,6,7,8

4.1 Using Matrices to Represent Data

Why *Matrices can be used to organize data. For example, information about picnic tables and barbeque grills can be organized into matrices.*

Objectives

- Represent mathematical and real-world data in a matrix.
- Find sums and differences of matrices and the scalar product of a number and a matrix.

APPLICATION
INVENTORY

The table below shows business activity for one month in a home-improvement store. The table shows stock (inventory on June 1), sales (during June), and receipt of new goods (deliveries in June).

2x6 matrix

	Inventory (June 1) Small	Inventory (June 1) Large	Sales (June) Small	Sales (June) Large	Deliveries (June) Small	Deliveries (June) Large
Picnic tables	8	10	7	9	15	20
Barbeque grills	15	12	15	12	18	24

column 1 col. 2
row 1
row 2

You can represent the inventory data in a *matrix*.

2x2 matrix

$$\text{Inventory matrix} \rightarrow \begin{array}{r} \\ \text{Picnic tables} \\ \text{Barbeque grills} \end{array} \begin{array}{c} \text{Small} \quad \text{Large} \\ \begin{bmatrix} 8 & 10 \\ 15 & 12 \end{bmatrix} \end{array} = M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$

A **matrix** (plural, *matrices*) is a rectangular array of numbers enclosed in a single set of brackets. The dimensions of a matrix are the number of horizontal rows and the number of vertical columns it has. For example, if a matrix has 2 rows and 3 columns, its dimensions are 2×3, read as "2 by 3." The inventory matrix above, M, is a matrix with dimensions of 2×2.

Dimensions
#rows "by" # columns

m_{21}
2nd row 1st column

Each number in the matrix is called an **entry**, or element. You can denote the *address* of the entry in row 2 and column 1 of the inventory matrix, M, as m_{21} and state that $m_{21} = 15$. This entry represents 15 small barbeque grills in stock on June 1.

EXAMPLE 1 **Represent the June sales data in matrix S. Interpret the entry at s_{12}.**

S_{12} =
matrix S
row 1
column 2

SOLUTION

$$\text{Sales matrix} \rightarrow \begin{array}{r} \\ R_1 \quad \text{Picnic tables} \\ \text{Barbeque grills} \end{array} \begin{array}{c} \quad\; C_2 \\ \text{Small} \quad \text{Large} \\ \begin{bmatrix} 7 & 9 \\ 15 & 12 \end{bmatrix} \end{array} = S$$

In matrix S, $s_{12} = 9$. In June, 9 large picnic tables were sold.

TRY THIS Represent the delivery data in matrix D. Interpret the entry at d_{21}.

$$D = \begin{array}{r} \\ \text{PicnicTables} \\ \text{BBQ grills} \end{array} \begin{array}{c} \text{Small} \quad \text{Large} \\ \begin{bmatrix} 15 & 20 \\ 18 & 24 \end{bmatrix} \end{array}$$

Integret D_{21} =

Addition and Scalar Multiplication

To find the sum (or difference) of matrices A and B with the same dimensions, find the sums (or differences) of *corresponding* entries in A and B.

EXAMPLE 3 Let $A = \begin{bmatrix} -2 & 0 & 1 \\ 5 & -7 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 7 & -1 \\ 0 & 2 & -8 \end{bmatrix}$.

a. Find $A + B$. **b.** Find $A - B$.

Dimensions

$A : 2 \times 3$

$B : 2 \times 3$

$$A - B = \begin{bmatrix} -2 & 0 & 1 \\ 5 & -7 & 8 \end{bmatrix} - \begin{bmatrix} 5 & 7 & -1 \\ 0 & 2 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} -2-5 & 0-7 & 1+1 \\ 5-0 & -7-2 & 8+8 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & -7 & 2 \\ 5 & -9 & 16 \end{bmatrix}$$

TRY THIS Let $A = \begin{bmatrix} 0 & 0 \\ 4 & 1 \\ -3 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} -10 & 5 \\ 0 & 4 \\ -7 & 3 \end{bmatrix}$.

a. Find $A - B$. **b.** Find $A + B$.

$$A - B = \begin{bmatrix} 10 & -5 \\ 4 & -3 \\ 4 & -8 \end{bmatrix}$$

$$A + B = \begin{bmatrix} -10 & 5 \\ 4 & 5 \\ -10 & -2 \end{bmatrix}$$

CHECKPOINT ✓ Is it possible to find the sum $\begin{bmatrix} -2 & 5 & 6 \\ 1 & -8 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -5 \\ 8 & -6 \\ -3 & 0 \end{bmatrix}$? Explain.

2×3 $\quad$ 3×2

Not the same dimensions
we cannot add them!

Refer to the table of business activity at the beginning of the lesson. Let matrices M, S, and D represent the inventory, sales, and delivery data, respectively.

Find $M - S + D$. Interpret the final matrix.

$A - B + C$

SOLUTION

$$M - S + D = \begin{bmatrix} 8 & 10 \\ 15 & 12 \end{bmatrix} - \begin{bmatrix} 7 & 9 \\ 15 & 12 \end{bmatrix} + \begin{bmatrix} 15 & 20 \\ 18 & 24 \end{bmatrix}$$

$$= \begin{bmatrix} 8-7+15 & 10-9+20 \\ 15-15+18 & 12-12+24 \end{bmatrix}$$

$$= \begin{array}{cl} \begin{array}{cc} \text{Small} & \text{Large} \end{array} & \\ \begin{bmatrix} 16 & 21 \\ 18 & 24 \end{bmatrix} & \begin{array}{l} \textbf{Picnic tables} \\ \textbf{Barbeque grills} \end{array} \end{array}$$

At the end of June, the store has 16 small and 21 large picnic tables in stock. It also has 18 small and 24 large barbeque grills.

Properties of Matrix Addition

For matrices A, B, and C, each with dimensions of $m \times n$:

Commutative	$A + B = B + A$
Associative	$(A + B) + C = A + (B + C)$
Additive Identity	The $m \times n$ matrix having 0 as all of its entries is the $m \times n$ identity matrix for addition.
Additive Inverse	For every $m \times n$ matrix A, the matrix whose entries are the opposite of those in A is the additive inverse of A.

To multiply a matrix, A, by a real number, k, write a matrix whose entries are k times each of the entries in matrix A. This operation is called **scalar multiplication**.

Let $A = \begin{bmatrix} 3 & 2 & 0 \\ -1 & -3 & 6 \\ 2 & 0 & -10 \end{bmatrix}$. Find $-2A$.

SOLUTION

$$-2A = \begin{bmatrix} -2(3) & -2(2) & -2(0) \\ -2(-1) & -2(-3) & -2(6) \\ -2(2) & -2(0) & -2(-10) \end{bmatrix} = \begin{bmatrix} -6 & -4 & 0 \\ 2 & 6 & -12 \\ -4 & 0 & 20 \end{bmatrix}$$

Where is scalar multiplication often used?

Geometric Transformations

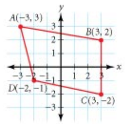
Example 6 shows you how to represent a polygon in the coordinate plane as a matrix.

6 **Represent quadrilateral $ABCD$ as matrix P.**

SOLUTION

Because each point has 2 coordinates and there are 4 points, create a 2×4 matrix.

$$\begin{array}{cccccc} & A & B & C & D & \\ P = & \left[\begin{matrix} -3 \\ 3 \end{matrix}\right. & \begin{matrix} 3 \\ 2 \end{matrix} & \begin{matrix} 3 \\ -2 \end{matrix} & \left.\begin{matrix} -2 \\ -1 \end{matrix}\right] & \begin{matrix} x\text{-coordinates} \\ y\text{-coordinates} \end{matrix} \end{array}$$



When you perform a *transformation* on one geometric figure to get another geometric figure, the original figure is called the **pre-image** and the resulting figure is the **image.** When you apply scalar multiplication to a matrix that represents a polygon, the product represents either an enlarged image or a reduced image of the pre-image polygon. This is shown in Example 7 on page 220.

7 Refer to quadrilateral $ABCD$ and matrix P in Example 6.

Graph the polygon that is represented by each matrix.

a. $2P$ **b.** $\frac{1}{2}P$

SOLUTION

a. Let quadrilateral $A'B'C'D'$ represent the image.

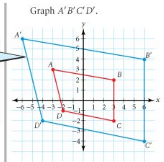
$$\begin{array}{ccccc} & A' & B' & C' & D' \\ 2P = & \left[\begin{matrix} -6 \\ 6 \end{matrix}\right. & \begin{matrix} 6 \\ 4 \end{matrix} & \begin{matrix} 6 \\ -4 \end{matrix} & \left.\begin{matrix} -4 \\ -2 \end{matrix}\right] \end{array}$$

b. Let quadrilateral $A''B''C''D''$ represent the image.

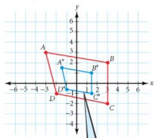
$$\begin{array}{ccccc} & A'' & B'' & C'' & D'' \\ \frac{1}{2}P = & \left[\begin{matrix} -\frac{3}{2} \\ \frac{3}{2} \end{matrix}\right. & \begin{matrix} \frac{3}{2} \\ 1 \end{matrix} & \begin{matrix} \frac{3}{2} \\ -1 \end{matrix} & \left.\begin{matrix} -1 \\ -\frac{1}{2} \end{matrix}\right] \end{array}$$

Graph $A'B'C'D'$.

Quadrilateral $A'B'C'D'$ is an enlarged image, or dilation, of quadrilateral $ABCD$ by a scale factor of 2.



Graph $A''B''C''D''$.



Quadrilateral $A''B''C''D''$ is a reduced image of quadrilateral $ABCD$ by a scale factor of $\frac{1}{2}$.

Working with Matrices

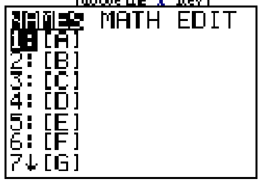
> Matrices are rectangular arrays of elements.
> The *dimension* of a matrix is the number of rows by the number of columns.

Adding Matrices - matrices must be of the *same dimension* to be added.

$$\text{Add: } \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 2 & 4 \\ 3 & 3 & 2 \end{bmatrix}$$

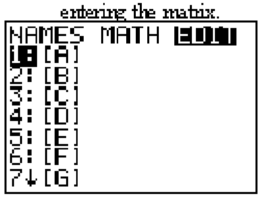
First Enter the Matrices (one at a time):

Step 1: Go to **Matrix** (above the x^{-1} key)

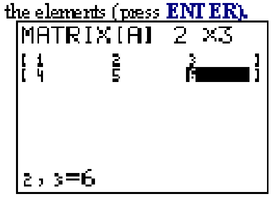


If dimensions appear next to the names of the matrices, such as 3x3, a matrix is already stored in the calculator. You may save it by moving to a new name, or overwrite it.

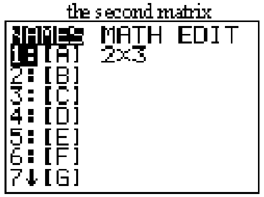
Step 2: Arrow to the right to **EDIT** to allow for entering the matrix.



Step 3: Type in the dimensions (size) of your matrix and enter the elements (press **ENTER**).

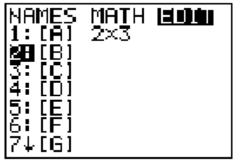


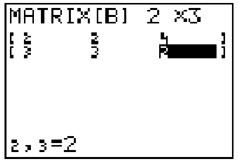
Step 4: Repeat this process for the second matrix



Step 5: Arrow to the right to **EDIT** and choose a new name.

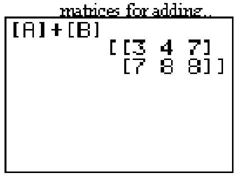
Step 6: Type in the dimensions (size) of your matrix and enter the elements (press **ENTER**).





Now, add:

Step 7: Return to the home screen. Go to **Matrix** to get the names of the matrices for adding.



The answer to the addition, as seen on the calculator screen, is =

$$\begin{bmatrix} 3 & 4 & 7 \\ 7 & 8 & 8 \end{bmatrix}$$

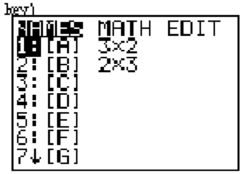
Multiplying Matrices

- for multiplication to occur, the *dimensions* of the matrices must be related in the following manner: $m \times n$ **times** $n \times r$ **yields** $m \times r$

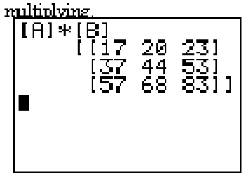
$$\text{Multiply: } \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \bullet \begin{bmatrix} 3 & 4 & 7 \\ 7 & 8 & 8 \end{bmatrix}$$

First Enter the Matrices (one at a time) as shown above:

Step 1: Once the matrices are entered, you should see their dimensions in residence when you go to **Matrix** (above the x^{-1} key)



Step 2: Return to the home screen. Go to **Matrix** to get the names of the matrices for multiplying.



The product, as seen on the calculator screen, is =

$$\begin{bmatrix} 17 & 20 & 23 \\ 37 & 44 & 53 \\ 57 & 68 & 83 \end{bmatrix}$$

Using Matrices to Solve Systems of Equations:

1. *(using the inverse coefficient matrix)*

Write this system as a matrix equation and solve: $3x + 5y = 7$ and $6x - y = -8$

Step 1: Line up the x, y and constant values.

$$\begin{aligned} 3x + 5y &= 7 \\ 6x - y &= -8 \end{aligned}$$

Step 2: Write as equivalent matrices.

$$\begin{bmatrix} 3x & 5y \\ 6x & -y \end{bmatrix} = \begin{bmatrix} 7 \\ -8 \end{bmatrix}$$

Step 3: Rewrite to separate out the variables.

$$\begin{bmatrix} 3 & 5 \\ 6 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -8 \end{bmatrix}$$

Step 4: Enter the two numerical matrices in the calculator.

```
NAMES MATH EDIT
1:[A]  2×2
2:[B]  2×1
3:[C]
4:[D]
5:[E]
6:[F]
7↓[G]
```

Step 5: The solution is obtained by multiplying both sides of the equation by the inverse of the matrix which is multiplied times the variables.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 6 & -1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 7 \\ -8 \end{bmatrix}$$

Step 6: Go to the home screen and enter the right side of the previous equation.

```
[A]⁻¹*[B]
                [[-1]
                 [2 ]]
```

The answer to the system, as seen on the calculator screen, is $x = -1$ and $y = 2$.

2. *(using Gauss-Jordan elimination method with reduced row echelon form)*
Solve this system of equations:

$$\begin{aligned} 2x - 3y + z &= -5 \\ 4x - y - 2z &= -7 \\ -x + 2z &= -1 \end{aligned}$$

Step 1: Line up the variables and constants

$$\begin{aligned} 2x - 3y + z &= -5 \\ 4x - y - 2z &= -7 \\ -x + 0y + 2z &= -1 \end{aligned}$$

Step 2: Write as an augmented matrix and enter into calculator.

$$\left[\begin{array}{cccc} 2 & 3 & 1 & 5 \\ 4 & 1 & 2 & 7 \\ 1 & 0 & 2 & 1 \end{array}\right]$$

Step 3: From the home screen, choose the **rref** function. [Go to **Matrix** (above the x^{-1} key), move right→**MATH**, choose **B: rref**]

```
NAMES MATH EDIT
6↑randM(
7:augment(
8:Matr▸list(
9:List▸matr(
0:cumSum(
A:ref(
B:rref(
```

Step 4: Choose name of matrix

Step 5: **The answer to the**

and hit **ENTER**.

```
rref([A])
   [[1 0 0  -3]
    [0 1 0  -1]
    [0 0 1  -2]]
```

system, will be the last column on the calculator screen:

$x = -3$
$y = -1$
$z = -2$.

Finding Your Way Around TABLE of CONTENTS